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MODELING THE DIVIDEND-PRICE RATIO: THE ROLE OF FUNDAMENTALS USING A REGIME-SWITCHING APPROACH

Steen Nielsen Jan Overgaard Olesen

Department of Economics - Copenhagen Business School
Solbjerg Plads 3, DK-2000 Frederiksberg

Modeling the Dividend-Price Ratio:

The Role of Fundamentals Using a Regime-Switching Approach *

by

Steen Nielsen & Jan Overgaard Olesen
(sn.eco@cbs.dk) (jo.eco@cbs.dk)

Department of Economics and EPRU
Copenhagen Business School
Solbjerg Plads 3 (5th)
DK-2000 Frederiksberg
Denmark

Abstract

Using annual data over the post-World War I-period, we estimate a fundamentals-based empirical model for the dividend-price ratio of Danish stocks. The key fundamentals-variable is a time-varying discount rate, decomposed into time-varying measures for the growth-adjusted real interest rate and the risk premium on stocks. In addition, the model includes real dividends and the lagged dividend-price ratio as explanatory variables. Results show that the model suffers from structural breaks over the sample. Using a two-state regime-switching approach to capture non-modeled shifts in the economic environment, we find that all fundamentals are highly significant in at least one regime and, moreover, obtain a good fit. The model identifies two very persistent regimes characterized by a 'low', respectively, 'high' dividend-price ratio.

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1. Introduction

In empirical finance the dividend-price ratio, defined as the ratio between a given periods dividend payments per share and the end-of-period stock price per share, is often - explicitly or implicitly - used as an indicator of whether stock prices are (too) high or (too) low. For instance, Campbell and Sheller (1998) report a very gloomy prediction for the US stock market based on the fact that the dividend-price ratio has fallen far below its historical mean, suggesting an overvalued stock market. Fama and French (1988) and Hodrick (1992) are other examples of the numerous studies that use dividend-price to forecast future stock returns, see also the survey in Campbell *et al.* (1997, Chp. 7).

However, according to standard finance theory one should expect time variation in the dividend-price ratio as a result of changes in the underlying economic fundamentals, in particular changes in the (*ex ante*) real interest rate and the risk premium on stocks relative to bonds. Hence, it is crucial to consider the economic fundamentals when using the dividend-price ratio to judge whether stocks are fairly valued or not. For this purpose we need an economic model for dividend-price. This is the topic of the present paper. Motivated by a Gordon growth type model which is modified to incorporate a time-varying discount rate, we formulate an empirical model using a real interest rate proxy, a proxy for the risk premium and the level of real dividend payments as explanatory variables. The real interest rate and risk premium proxies together capture the effects from the time-varying discount rate while the inclusion of real dividends allows for the possibility that innovations in dividends are reflected less than proportionately in stock prices. We also include lagged dividend-price in the model to allow for slow adjustment in the dividend-price ratio to long-run equilibrium.

The economic model is estimated for the aggregate Danish stock market, using annual observations for the period 1927-1996. All variables turn out to be significant with the right signs and a reasonably good fit is obtained. However, the model suffers from structural breaks as the coefficients to the explanatory variables are highly unstable. This suggests that we have omitted an important (or several important) fundamental variable(s). In the Danish case, a possible explanation for a structural break is a change in investor taxation as of 1983, i.e., the introduction of a separate pension fund tax on bond investments, affecting the relative profitability of stock investments (cf. below). Modifying the economic framework in order to take account of the omitted variable is obviously the ideal solution in such a situation. However, in practice this may not always be realistic or even possible because the omitted variable may be difficult (or impossible) to identify and, subsequently, quantify. When modeling the effects of investor taxes in a heterogeneous tax system as the Danish where tax rates differ significantly across investor groups, it is essential to correctly identify, at every single point of time, the 'marginal investor', defined as the stock investor who has a reservation price or willingness-to-pay for stocks which at the margin is equal to the prevailing stock price. However, the 'marginal investor' is unobservable and, hence, the inclusion of investor taxes in the model is a difficult task. In the case of the new pension fund tax, matters are, moreover, complicated by the gradual implementation of the tax.

In this paper, we take a short-cut by estimating the economic model using the two-state regime-switching approach of Hamilton (1990). We consider this approach to be a practical tool of incorporating and indirectly modeling the omitted factor(s) which give rise to the structural breaks that we encounter in the one-regime specification, without having to explicitly model those factors. The regime-switching approach is based on the assumption that the economic model differs across (a finite number of) distinct regimes, whose timing is governed by an exogenous, discrete and latent state-variable. This means that the type of omitted factors which we can capture by this approach are the more persistent factors that relate to the 'economic environment' of the model and that result in the outcome of distinct regimes over time with distinct economic models. Such factors often relate to the institutional or policy framework of the economy, leading to distinct policy or institutional regimes over time, and are typically also the factors that are difficult to model. We find in our case that the regimes identified by the regime-switching approach are highly persistent which is consistent with the interpretation that the omitted factor(s) represents changes in the economic environment rather than being an additional temporary explanatory variable. In particular, we conjecture that the identified regimes may be given the interpretation of different tax policy regimes.

Beyond providing a practical modeling tool, we also consider the analysis based on the regime-switching approach to be a useful step in identifying the possible omitted factor(s) because the results provide valuable insight regarding the timing of regime-shifts, without being conditioned on *apriori* information. Hence, the regime-switching model lets the data determine if and when regime shifts occur. This information can consequently be used to identify candidates for omitted factors by examining relevant institutional or policy changes around these dates of regime-shifts.

The regime-switching approach of Hamilton (1990) has previously been used in the empirical literature to model asset pricing in situations where the pricing process changes over time, e.g. due to shifts in the process governing economic fundamentals (for instance as a result of policy regime shifts), shifts in the predominance of different investor types over time or changes in the institutional set up or taxation rules of relevance for the stock market. The importance of regime-shifts in the pricing process has recently been emphasized for the US stock market by Driffill and Sola (1998) who motivate shifts in the pricing process with regime-shifts in the underlying process for dividends, cf. the discussion at the end of this paper. The possible influence of different investor types with different investment rules has been examined for the currency market by Vigfusson (1996), who assumes that the market on a high-frequency (daily) basis shifts between being driven by chartists and fundamentalists. In the context of the stock market, a potential motivation for time differences in investment and, hence, pricing rules could be that the market misprices stocks in high-inflation regimes by using nominal rather than real interest rates, whereas investors may price stocks more correctly in low-inflation regimes, cf. Modigliani and Cohn (1979), who argue that US stocks were mispriced (undervalued) in the high-inflation regime of the 1970s. In such a setting we should *apriori* expect the regimes identified by the regime-switching approach to be identical to different inflationary regimes. In this paper, we do not attempt at formally explaining the regime shifts but the working hypothesis motivating the use of the regime-switching approach is that the regime shifts are related to (persistent) changes in the ‘economic environment’, leading to (persistent) shifts in the economic model linking dividend-price to the economic fundamentals. We conjecture that changes in investor taxation is a prime candidate but institutional changes or changes in the processes for the economic fundamentals leading to changes in expectations formation and hence the economic model may do as well[1]. In any case, a closer examination of the causes underlying the regime shifts would be interesting but this is left for future work.

Results from estimating the regime-switching model show that all the fundamentals variables including the real interest rate and the risk premium are highly significant in at least one regime. Hence, we have succeeded in modeling a time-varying discount rate, here decomposed into a time-varying real interest rate and a time-varying risk premium, which is empirically significant in explaining the dividend-price ratio. This is an innovation compared to the existing empirical literature where the discount rate is either assumed to be fixed or not quantified directly (no closed-form measure) when modeling the behavior of the dividend-price ratio or, more generally, stock prices, cf. e.g. Driffill and Sola (1998), Froot and Obstfeld (1991) and Campbell *et al.* (1997, Chp. 7). Our model is not perfect in terms of misspecification tests but passes at a 5% significance level, is stable over time and provides a rather good fit to dividend-price. Moreover, results show that two regimes are both necessary and sufficient to remove the structural breaks from the underlying economic model. The model clearly identifies 3 distinct subperiods over which the regimes reign (1927-1949,1950-1985 and 1986-1991), thereby providing valuable insight which can be used in inferring the possible causes of the two regimes.

The outline of the paper is as follows. In section 2, we formulate an operational empirical model, based on a simple and *ad hoc* theoretical framework which is derived from the standard Gordon growth model by allowing for a time-varying discount rate. The data is reviewed in section 3. In section 4 we, first, estimate the economic model under the assumption that only one regime applies, i.e., assuming that the model is stable over the entire sample. In section 5, we estimate the regime-switching model allowing for two distinct regimes. Section 6 finally concludes the paper.

[1] To illustrate, consider a change in the process for the real interest rate leading to increased short-run volatility. This may imply that investors put less emphasis on the current level of the real interest rate when forming expectations about the future long-run, average real interest rate, which is the relevant measure for the pricing of stocks. The implication is a change in the economic model with a smaller coefficient to the current real interest rate.

2. The Empirical Model

In formulating the empirical model, we take as a starting point the textbook Gordon growth model for the price of a stock with a constant discount rate and constant expected dividend-growth, see Gordon (1962) or Campbell *et al.* (1997). We modify Gordon´s model in a rather simple way to allow for time variation in the discount rate, reflecting time variation in both the real interest rate and the risk premium on stocks. The resulting theoretical framework is *ad hoc* but allows us to formulate an operational empirical model with specific candidates for economic variables that may explain dividend-price. The theoretical model can, basically, be interpreted as an assumption that market participants at each point in time price stocks according to the constant-discount-rate-and-constant-dividend-growth Gordon model, i.e., as if the discount rate and dividend-growth were in fact constant, while using the prevailing levels for nominal bond returns, expected nominal dividend growth and the risk premium on stocks as determinants[2].

Thus, let the equilibrium in stock and bond markets at each point in time t be described by a no-arbitrage relation stating that the expected (nominal) return on stocks $E_t[S_{t+1}]$ from time t to t+1 should be equal to the corresponding (nominal) return on bonds B_t augmented by a risk premium γ_t on stocks relative to bonds:

(1) $$E_t\left[S_{t+1}\right] = B_t + \gamma_t$$

We take B_t to be the yield-to-maturity on a (one-period) bond so that it is predetermined and known as of time t.

The return on stocks is given as the sum of capital gains and dividend yield:

[2] Campbell and Shiller (1988) have generalized Gordon's growth model to take account of a stochastic, time-varying discount rate, the so-called "dynamic Gordon growth model". However, their model is (at least in its general version) not as operational as the one we formulate. In particular, the Campbell and Shiller (1988) model does not entail a closed-form expression for the time-varying discount rate. Our assumptions on expectations formation, basically, imply that stocks can be priced within the original Gordon model despite the fact that the discount rate (and dividend growth) vary over time.

(2) $$E_t\left[S_{t+1}\right]=\frac{E_t\left[P_{t+1}\right]-P_t}{P_t}+\frac{E_t\left[D_{t+1}\right]}{P_t}$$

where P_t is the *ex dividend* price per share as of time t (i.e., at the beginning of period t+1) and D_{t+1} is the dividend payment per share paid during period t+1.

Even though B_t and γ_t are allowed to vary stochastically over time, we shall assume that market participants only form *point* expectations wrt. future bond returns and risk premia, i.e., “Certainty Equivalence” is assumed to apply. Moreover, we assume that market participants expect bond returns and risk premia to be constant over time, so that any innovations in the two variables are expected to be permanent. These assumptions - while clearly restrictive in a theoretical setting - allow us to set up an empirically tractable model. Thus, under the additional Gordon assumption of constant expected dividend growth, (1) can be solved by forward recursion to give the following no-bubble solution for the dividend-price ratio (assuming that $R_t+\gamma_t>0$):

(3) $$\frac{D_t}{P_t}=\frac{R_t+\gamma_t}{1+G_t}\approx R_t+\gamma_t \quad , \quad where \quad R_t\equiv B_t-G_t$$

G_t is the expected *nominal* growth in dividends per share as of time t. G_t is also allowed to vary over time. According to (3), the dividend-price ratio is in equilibrium equal to the sum of the (*ex ante*) growth-adjusted real interest rate $R_t\equiv B_t-G_t$ and the risk premium on stocks γ_t. (3) resembles the solution of the standard Gordon growth model with the main difference being the allowed variation in the real interest rate and the risk premium and, hence, the discount rate (the sum of the two).

Based on (3), we set up the empirical model:

(4) $$\frac{D_t}{P_t}=\beta_0+\beta_1R_t+\beta_2\gamma_t+\beta_3DR_t+\beta_4\frac{D_{t-1}}{P_{t-1}}+\varepsilon_t$$

where ϵ_t is the residual of the equation. We have augmented the model by including the lagged dividend-price ratio $(D/P)_{t-1}$ and the log-level of real dividends per share DR_t as additional candidate explanatory variables. The introduction of the former allows for slow or partial adjustment in the dividend-price ratio so that (3) (or rather the long-run solution to (4)) is basically thought of as a model for the long run, providing us with an equilibrium relation to which dividend-price adjusts in the long run. The introduction of DR_t allows for the possibility that real stock prices may react more or less than proportional to innovations in real dividend payments. According to (3), the relation between real stock prices and real dividends should be proportional as the dividend-price ratio is unaffected by innovations in dividends. The reason is that market participants expect any innovation in current dividends to be permanent under the Gordon constant-dividend-growth setting. However, this may not be the case empirically. Froot and Obstfeld (1991) and Driffill and Sola (1998) also include real dividends in their models for the price-dividend ratio (the inverse of the dividend-price) in order to capture the possibility of "intrinsic bubbles" in stock prices, i.e., rational bubbles that depend on fundamental variables. As standard in empirical analysis, we allow for a constant term in (4), even though not strictly implied by the theoretical model. Hence, we intend to explain the variations in rather than the actual levels of the dividend-price ratio[3].

The challenge facing (4) is the fact that the real interest rate R_t and the risk premium γ_t are unobservable. We, therefore, have to use suitable proxies for these two variables.

3. The Data

The data are depicted in Figures 1-4. The source database is Nielsen, Olesen and Risager (1999) which comprises data for the Danish stock and bond markets. Stock market data relate to the aggregate market level of all Danish firms listed at the Copenhagen Stock Exchange. The market index by Statistics Denmark is used for stock prices while dividend payments are estimated from a large sample of firms, cf. Nielsen, Olesen and Risager (1999) for further details. Bond data relate to the markets for government bonds. All observations are annual.

[3] Note that according to the constant-discount-rate Gordon model, all regressors in (4) should be insignificant, implying that the dividend-price ratio would, basically, follow a white noise process around a constant mean. However, the latter assumption is clearly violated by the data on dividend-price, cf. below, as we observe systematic deviations in the ratio from its mean. It is these systematic deviations that we intend to explain.

The empirical analysis in the following sections uses the sample period 1927-1991 which is the longest available sample for all variables.

< Insert Figures 1-4 around here >

Figure 1 shows the dividend-price ratio over the period 1927-1996. The plot suggests a cyclical component in the ratio with large and often persistent deviations from its sample mean, in particular, in the first half of the period. For instance, stock prices seem to have been persistently low compared to dividends in the first half of the 1950s while stock prices were high during World War II. In relative terms, the ratio is often subject to large year-by-year changes where in particular the drop in the ratio from 5.2% in 1982 to 1.8% in 1983 (a decrease of 65% in relative terms) attracts attention. This fall which is a result of capital gains on stocks of 114% that year coincides with at least two important events in the Danish economy. First of all, there was a major shift in economic policy as a new conservative-liberal government came into office in September 1982, emphasizing tight economic policies including a fixed exchange rate policy. Second, a new tax was introduced on the returns on pensions funds' bond holdings while the returns on stocks were exempted from taxation[4]. This *ceteris paribus* gave pension funds an incentive to invest more in stocks and less in bonds. We can also observe that the dividend-price ratio has been at a historically low level since 1983. The post-1983 average is 1.7% which compares to an average of 5.1% over the years before 1983. This persistent low level is a key issue in understanding the mechanisms which determine the dividend-price ratio and it is, in particular, of interest to know whether it can be explained by economic fundamentals or whether it marks a new regime compared to the pre-1983 history.

The proxy used for the latent growth-adjusted real interest rate R_t is plotted in Figure 2. The real interest rate as of time t is constructed as the 5-year yield-to-maturity on government bonds at time t minus the realized growth in *nominal* dividends over the corresponding 5-year period following time t. The proxy is an *ex post (or perfect foresight)* growth-adjusted real interest rate where the use of growth in nominal dividends to take account of inflation and real growth is consistent with the theoretical framework of section 2. Because of the forward-looking nature of the proxy, we loose 5 observations towards the end of the period so that the sample effectively ends in 1991[5].

It is evident from Figure 2 that the real interest rate proxy is highly volatile. The fluctuations are mainly driven by the variation in the dividend growth part of the proxy, as the 5-year nominal bond return is much more stable on a year-by-year basis. For instance, the low levels of the real interest rate in the 1940s is due to high future dividend growth which is not accompanied by high nominal bond returns. Due to a non-credible stance of economic policy (amongst other things), the Danish economy experienced very high nominal and real interest rates towards the end of the 1970s and in the beginning of the 1980s. Nominal interest rates declined following the new policy regime as of late 1982, but nominal dividend growth declined likewise, sustaining the high real interest rate level until the end of the 1980s.

To construct a proxy for the risk premium on stocks γ_t, we draw on Olesen and Risager (1999) who test whether the Danish premium on stocks, defined as the excess of stock returns over bond returns, can be predicted from a set of possible predictor variables such as the dividend price ratio, dividend yield, bond returns, lagged equity premia etc. They conclude that the 5-year premium on stocks is predictable from the dividend yield, the 5-year bond return and past 1-year equity premia, see Olesen and Risager (1999) for details. This predicted or fitted 5 year premium can in an efficient markets framework be interpreted as an estimate of the risk premium on stocks relative to bonds. However, Olesen and Risager (1999) use the dividend yield as a predictor, and the dividend yield comes close to the dividend-price ratio variable. In terms of (4), one could therefore possibly argue that when using the fitted premium in Olesen and Risager (1999) as the risk premium proxy γ_t, we

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[4] This tax was passed by the Parliament in 1983 and took effect as of Jan 1 1984. Because pension savings before 1984 were exempted from taxation, the tax was phased in gradually.

[5] Data for bond yields are available for the 1-, 5- and 10-year maturities, cf. Nielsen, Olesen and Risager (1999). In choosing between these horizons, we excluded the latter because using a 10-year *ex post* real interest rate would imply a loss of more observations towards the end of the sample period. We excluded the 1-year horizon because the resulting 1-year real interest rate turned out to be a very ‘noisy’ measure with large year-to-year variability and no explanatory power wrt. the dividend-price ratio. Intuitively, the 1-year maturity also appears to be too short in order to be of relevance for the pricing of stocks in practice.

would basically explain the dividend-price ratio by a variable that comes close the ratio itself, the dividend yield.

In order to be immune to this critique, we have therefore (re-)estimated a predictor model excluding the dividend yield and the dividend-price ratio as potential predictor variables. Following the approach of Olesen and Risager (1999), the resulting model is (standard errors of coefficient estimates in parentheses)[6]:

(5) $$\hat{PR5}_t = \underset{(0.727)}{2.804} - \underset{(0.023)}{0.113} * PR1_{t-1} - \underset{(0.037)}{0.106} * PR1_{t-2} - \underset{(0.030)}{0.093} * PR1_{t-3}$$

$PR1_t$ and $PR5_t$ are the equity premia, calculated as the simple difference between stock and bond returns, over the 1-year, respectively, 5-year holding period starting at time t. $\hat{PR5}_t$ is the 5-year premium predicted or fitted from the model. According to (5), the premium on stocks over the next 5-year period can be predicted from the preceding three years' (known) realizations of 1-year equity premia. The coefficients are negative which suggests a significant mean-reverting component in stock prices. We use (5) as the proxy for the risk premium, i.e., $\gamma_t \equiv \hat{PR5}_t$ [7]. Notice that from (1), the risk premium γ_t should be equal to the predicted premium on stocks so our proxy is consistent with the theoretical framework.

γ_t is plotted in Figure 3. The risk premium proxy is also highly volatile, in particular, towards the end of the sample. The large drop in the risk premium in the beginning of the 1980s partially coincides with the shift in the economic policy regime, cf. above. The large negative risk premia in the years 1983-1985 may possibly (to some extent) be explained by the

[6] (5) is estimated according to a 'general-to-specific' procedure by, first, estimating a full model where the 5-year premium is regressed on all candidate predictor variables (bond returns, term structure components, past 1-year equity premia) and, subsequently, removing all insignificant predictors successively, using a 5% significance level. (5) is the resulting parsimonious model. All parameters are estimated by OLS while Newey-West standard errors which are consistent to heteroskedasticity and serial correlation in the disturbance term (up to lag 5) are used for standard errors of the coefficient estimates. The sample is the available period 1927-1992, using overlapping observations. (5) explains 36% (=R^2) of the variation in the actual 5-year equity premium. The residual has a standard deviation of 4.5%. Notice that we differ from Olesen and Risager (1999) by using absolute rather than logarithmic returns.

[7] The predictions of (5) come close to those reported in Olesen and Risager (1999), in particular, as regards the significant movements and turning points in the 5-year equity premium.

introduction of the new pension fund tax on bond returns as we *ceteris paribus* should expect pension funds to demand a smaller before-tax 'risk' premium on stocks relative to bonds under the new tax regime. To see this, note that with the pension fund tax the no-arbitrage relation between stock and bond returns is modified from (1) to:

$$(1') \qquad E_t\left[S_{t+1}\right] = (1-\tau)B_t + \gamma_t^*$$

where we have assumed that a pension fund is the (representative) marginal investor. τ is the pension fund tax on bond returns, $(1-\tau)B_t$ is the after-tax bond return and γ_t^* denotes the ('pure') risk premium on stocks. From (1'), the before-tax premium on stocks γ_t, i.e., the expected before-tax excess return on stocks $\gamma_t \equiv E_t(S_{t+1})-B_t$, is related to the after-tax premium γ_t^* by $\gamma_t = -\tau B_t + \gamma_t^*$. Thus, the introduction of the pension fund tax *ceteris paribus* lowers the before-tax premium. For sufficiently high bond returns B_t - and bond returns were still high in the years 1983-1985 - we may even expect a negative before-tax premium.

Notice that we could, in principle, have constructed a proxy for the 'pure' risk premium γ_t^* if we had known the relevant tax rate τ for each year in the sample. However, constructing a data series for τ is impeded both by interim arrangements for the pension fund tax and by the fact that we need to know the relevant but latent 'marginal investor' (which we do not). These complications motivate our use of the 'before-tax' proxy γ_t [8] [9].

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[8] We proceed by calling γ_t a 'risk premium' even though this is not entirely adequate in the presence of the pension fund tax. Thus, with the tax, γ_t captures both the 'true' risk premium γ_t^* and the distortionary tax effect $-\tau B_t$.

[9] We can also introduce taxes in the theoretical framework of section 2. Using (1') instead of (1), the with-tax solution for the dividend-price ratio is:

$$(3') \qquad \frac{D_t}{P_t} \approx (1-\tau)B_t - G_t + \gamma_t^* = B_t - G_t + \gamma_t$$

where the final equation follows from the relationship between γ_t and γ_t^*. (3') is actually identical to the without-tax solution in (3). Thus, in terms of the theoretical framework, the introduction of the pension fund tax does not matter, i.e., it does not change the solution (the structural equation) for dividend-price. The reason is that we include the before-tax 'risk premium' γ_t which fully incorporates the stock price effects of the new tax. Note, however, that the pension fund tax *ceteris paribus* lowers the level of dividend-price by lowering γ_t. Moreover, it is crucial for the result that the real interest rate and the risk premium have the same quantitative effects on dividend-price. Thus, allowing for taxes in the empirical model (4) (replacing R_t and γ_t with the after-tax real interest rate $(1-\tau)B_t-G_t$ and the 'true' risk premium γ_t^*, respectively, and rewriting) we get:

Both the real interest rate proxy and the risk premium proxy are negative in some of the years and the sum of the two proxies also turns out to be negative occasionally. The latter is obviously not consistent with the theoretical framework of section 2 which requires the discount rate $R_t+\gamma_t$ to be strictly positive in order to result in a well-defined (finite) forward-looking stock price solution. The estimation results to follow, however, suggest that market participants - in contrast with theory - expect a significant degree of mean reversion in the real interest rate and the risk premium so that negative values for the *current* real interest rate and the *current* risk premium may *apriori* be perfectly valid because it is expected to be a temporary phenomenon. Moreover, in terms of the empirical model (4), what matters are the variations in the real interest rate and the risk premium proxies over time (in which we may have more confidence) rather than the actual levels.

Finally, Figure 4 illustrates the log-level of real dividend payments. Dividends show some turbulence in the beginning and towards the end of the sample but have otherwise shown a steady declining trend.

4. Results Using a One-Regime Approach

Column 2 in Table 1 reports the results from estimating (4) over the entire sample 1927-1991, assuming that only one regime prevails. The estimation is performed by the Maximum Likelihood (ML) method under the assumption that the disturbance term of (4) is normal and independent distributed over time with homoskedastic variance ($\epsilon_t \sim Nid(0,\sigma^2)$). The ML coefficient estimates are identical to those obtained by Ordinary Least Squares (OLS).

(4') $$\frac{D_t}{P_t} = \beta_0 + \beta_1 R_t + \beta_2 \gamma_t + (\beta_2 - \beta_1)\tau B_t + \beta_3 DR_t + \beta_4 \frac{D_{t-1}}{P_{t-1}} + \varepsilon_t$$

The pension fund tax leaves the structural model unchanged iff the coefficients β_1 and β_2 are identical. If the coefficients differ, an additional explanatory variable τB_t, capturing the tax distortion, is introduced into the model. As the results will show, cf. below, the latter is the relevant case empirically and we should *apriori* expect a regime-shift in the empirical model at the time of the introduction of the new tax (because the additional variable is not included). Therefore, in a more general setting than (3), we can not be sure that the structural model for dividend-price will be unaffected by the pension fund tax and, hence, the question of whether or not the model survives becomes an empirical issue. The empirical results suggest that the inclusion of the pension fund tax in the risk premium construction γ_t does not sufficiently account for the effects of this tax on dividend-price, as we estimate a structural break in the model (a regime-shift) in the mid-1980s, cf. section 5 below.

< Table 1 >

Using the ML standard errors, all coefficients are highly significant. The real interest rate and the risk premium have the expected positive effects on the dividend-price ratio. The magnitudes are, however, less than predicted by theory. This applies both to the ‘short run effects’ (coefficients of 0.0345 and 0.1444, respectively, cf. Table 1) and the ‘long run effects’ (0.062 and 0.259, respectively)[10], where the latter take account of the evident slow adjustment in the dividend-price process, cf. below. According to the theoretical framework of section 2, we should expect a coefficient of one for both variables but this value falls far above the point estimates and the deviations are much larger than what the uncertainty of the coefficient estimates allows for. When inspecting Figures 1 through 3, the result is not surprising as the variation intervals for the real interest rate and the risk premium are much larger than for the dividend-price ratio. This result suggests, tentatively, that market participants do not expect innovations in the two variables to be permanent, as assumed in the theoretical framework, but that they expect some significant degree of mean reversion in the real interest rate and the risk premium so that current realizations of the variables receive (relatively) less weight[11]. The mean reversion feature seems perfectly reasonable from the time series behavior of the two variables, cf. Figures 2 and 3.

Real dividends also have a significant effect on dividend-price. The effect is positive, implying that an increase in real dividends gives rise to a less than proportional increase in real stock prices. This is, again, a deviation from theory and suggests that market participants do not consider innovations in dividends to be permanent but expect some degree of mean reversion.

Finally, the significance of lagged dividend-price indicates slow or partial adjustment in the dividend-price process to its long-run equilibrium.

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[10] By dividing through by one minus the autoregressive coefficient 0.4433 in the model of Table 1, the long-run equilibrium model for dividend-price becomes (ignoring the residual term):

$$\frac{D_t}{P_t} = -4.883 + 0.062 * R_t + 0.259 * \gamma_t + 2.308 * DR_t$$

[11] Of course, the result may also suggest that the proxies used for the real interest rate and the risk premium are too volatile. However, the high significance of the proxies validates their use.

Figure 5 illustrates the fit of the model. The one-regime model seems to work reasonably well and is, in particular, able to track the significant fall in dividend-price in 1983. There are, however, also episodes of systematic under- or overvaluation of dividend-price, see for instance the periods 1946-1956 and 1985-1991.

< Figure 5 >

The model passes the White and Lagrange Multiplier (LM) specification tests for serial correlation (at lag 1) and heteroskedasticity (ARCH) in the residual term, using conventional significance levels, see the bottom half of Table 1[12]. There is, however, strong evidence of serial correlation at higher lags (AR(3) and AR(5)) leading to a rejection of the model. Note that the documented serial correlation implies that the coefficient estimates are inconsistent, given the presence of the lagged dependent dividend-price as a regressor. The coefficients should therefore be interpreted with caution.

Another severe problem with the model is that is highly unstable over time. Figures 6-10 show recursive estimates of the model coefficients including 95% confidence bands, obtained by recursive least squares. With the exception of the risk premium, the coefficients are very unstable and there is a strong indication of a structural break in the model both in the beginning and towards the end of the sample.

< Figures 6-10 >

The apparent instability of the model can be further documented by formal testing. The Andrews test, see Table 1, allows one to perform a test for structural break without having to pre-specify a candidate time for a breakpoint, see Andrews (1993) and Hamilton (1996) for details. The Andrews test procedure basically performs a LM test for a shift in the mean at each point in the sample, except for the first 15% and the last 15% of the observations. One

[12] The tests are documented in Hamilton (1996). We use the suggested small-sample versions of the tests whereby the asymptotic test is transformed to a small-sample test based on the F-distribution. The tests for autocorrelation are tests for an AR(1) process in the residual term.

then chooses the observation with the highest LM test value and compares with critical test values, as tabulated in Andrews (1993). The evidence for the one-regime model is a clear indication of a (at least one) structural break in the sample. Hence, the Andrews (1993) test statistic is 23 which should be compared to critical values of 8.85 (5% significance level) and 12.35 (1%), that is, a clear rejection of the null hypothesis of no structural breaks. The test statistic is obtained for the year 1947[13].

To conclude, the estimation results suggest that we have identified economic fundamentals variables which have power in explaining the variations in the dividend-price ratio, including the large fall in 1983. There are, however, specification problems with the model and there is, in particular, strong evidence that the one-regime model is unstable over time, suggesting that more than one regime applies over the sample period.

5. Results Using a Regime-Switching Approach

Motivated by the evident instability of the economic model in (4), we estimate a version of the model which allows for more than one regime. A regime is here defined as a subperiod (or several subperiods) over which (4) is stable, i.e., over which the coefficients of the economic fundamentals (including the constant term) and the explanatory power of the model (as measured by the residual variance) are constant. A regime shift takes place whenever the underlying structural framework for dividend-price changes either because of a change in the impacts of the various fundamentals or because of a change in the non-explained part of the volatility in dividend-price. In other words, a regime shift can be interpreted as a structural break in the underlying economic model. We incorporate the possibility of multiple regimes by using the Markovian regime-switching model developed by Hamilton (1990). This approach has the advantage of letting the data - as opposed to *apriori* information - determine whether there is more than one regime and, if affirmative, when the regime shifts take place. In order to keep the model as simple as possible, we only allow for two regimes from the outset and, subsequently, test whether two regimes are sufficient to eliminate the structural breaks in (4).

[13] The individual LM test values over the sample are reported in the Appendix.

In the regime-switching approach, the economy can at each point of time be in one of two possible states, as indexed by an unobservable state-variable s_t which takes on the values 1 or 2 [14]. Each regime is described by a distinct model for the dividend-price ratio:

(6)

$$\frac{D_t}{P_t} = \beta_0(s_t) + \beta_1(s_t) * R_t + \beta_2(s_t) * \gamma_t + \beta_3(s_t) * DR_t + \beta_4(s_t) * \frac{D_{t-1}}{P_{t-1}} + \sigma(s_t) * \varepsilon_t \ , \quad s_t = 1, 2$$

where the parameters depend on the prevailing state s_t. (6) is identical to (4) except for the state-dependence so that the underlying economic framework is fundamentally unchanged. The crucial difference in (6) is that we here operate with (possibly) two distinct models which differ wrt. parameters, i.e., the coefficients $\beta_i(s_t)$ and the residual variance $\sigma(s_t)^2$.

Which model applies at a given point of time is governed by the state-variable s_t. s_t is stochastic and is assumed to follow a two-state Markov Chain with constant transition probabilities p_{ij}, where the latter is defined as the probability that the state (or regime) is *j* in period t conditional on the state *i* in period t-1, i.e., $p_{ij} \equiv Pr\{s_t=j|s_{t-1}=i\}$ $(i,j=1,2)$. s_t is by assumption independent of the residual term ϵ_t across all time periods, so that the state process is exogenous to the dynamics of dividend-price.

Under the assumption that ϵ_t is independent standard normal (ϵ_t~Nid(0,1)), we can estimate (6) by the method of Maximum Likelihood (ML), see Hamilton (1994, Section 22). The results are reported in columns 3 and 4 of Table 1[15].

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[14] For a detailed outline of the regime-switching model including the statistical foundations, we refer to Hamilton (1990), Hamilton (1996) or the textbook Hamilton (1994, Section 22). Numerous applications of the model can be found, including those in Driffill and Sola (1998), Engel and Hamilton (1990) and Hamilton and Lin (1996).

[15] We have performed the estimation under the assumption that the state probabilities of the initial observation are given by the ergodic probabilities. Including an estimation of the initial probabilities does not change the results significantly. The computations are performed with the BFGS algorithm in GAUSS using a variety of different starting values. We identify more than one local maximum (a total of 5) depending on the starting values and, moreover, encounter a singularity problem of the likelihood function, that is, for certain starting values the likelihood becomes ‘large’ without convergence as one of the regime-dependent variances goes to zero. The results of Table 1 apply to the local maximum with the highest likelihood. This choice is consistent with Kiefer (1978) who shows for the mixed-distribution model - where a global maximum does not exist - that there is a bounded local maximum of the likelihood function (with variances being positive) which exhibits the usual maximum likelihood properties of being consistent and asymptotically efficient.

First of all, we note that all coefficients have the expected signs. In regime #2, all coefficients can be shown to be significant at the 1% significance level, whereas the real interest rate and lagged dividend-price turn out to be insignificant in regime #1[16]. The two remaining variables (the risk premium and real dividends) are highly significant in regime #1. In fact, for the risk premium and real dividends, the coefficient estimates do not differ much across regimes. The results suggest that we have two regime-dependent, underlying models for dividend-price, one in which there is partial adjustment in dividend-price and where both the real interest rate, the risk premium and real dividends matter (regime #2), and one in which there is an immediate adjustment and where only the risk premium and real dividends are important (regime #1). Because the estimated residual variance is markedly higher in regime #2 than in regime #1, the uncertainty attached to the model's fit is larger in the former regime (despite the model having more significant explanatory variables in this regime)[17].

The autoregressive term in the dividend-price model, reflecting partial adjustment, implies that the impact of the fundamentals variables is (slightly) stronger in the long term than in the short term. This difference between the two horizons is most pronounced for regime #2 where the autoregressive term has the largest coefficient and the adjustment to long run equilibrium, hence, is the slowest. The long run equilibrium relation pertaining to each regime can be calculated from Table 1 by dividing through by one minus the autoregressive coefficient. This leads to (ignoring the error term)[18]:

[16] A Likelihood Ratio test of the joint hypothesis that the real interest rate and lagged dividend-price are insignificant in regime #1 gives a test statistic of 3.4 with two degrees of freedom, corresponding to a critical significance value of 18.4%. Hence, the hypothesis is accepted at conventional significance levels. We keep the two variables in the model because the resulting parsimonious model fails the specification tests.

[17] From a probabilistic inference, cf. below, we can estimate regime #2 to have reigned over the period 1950-1985. Using the model's overall fit (see equation (8) below), the coefficient of determination (R^2) over this subperiod is 81%. This is considerably lower than the R^2 of 96% in the remaining periods 1927-1949 and 1986-1991 (regime #1), indicating a lower explanatory power for the model in regime #2. Over the whole sample, the R^2 is 91%. Notice that the dating of regimes is not certain so the differences should be interpreted with caution.

[18] As shown in Nielsen and Olesen (1999), the computation of the regime-dependent mean $E[(D/P)_t \mid s_t]$ is complicated when allowing for an autoregressive dependent term. (7) should therefore correctly be interpreted as the expected dividend-price ratio conditional on being in regime #1, respectively regime #2 *both in the current and previous period*, i.e., as $E[(D/P)_t \mid s_t=s_{t-1}=i]$ (i=1,2). However, this mean will come close to that of $E[(D/P)_t \mid s_t]$ whenever the regimes are persistent, which turns out to be the case for our model.

(7)
$$\frac{D_t}{P_t} = -6.562 + 0.019 * R_t + 0.158 * \gamma_t + 2.676 * DR_t \qquad (regime\#1)$$
$$\frac{D_t}{P_t} = -6.319 + 0.094 * R_t + 0.199 * \gamma_t + 2.787 * DR_t \qquad (regime\#2)$$

The main difference between the two regimes is the real interest rate impact which is insignificant in regime #1. The impact of the risk premium is also somewhat larger in regime #2, whereas the coefficients to real dividends (and the constant terms) are almost equal across regimes.

As was also the case for the one-regime model, both the real interest rate and the risk premium have smaller effects than expected *apriori*, that is, the coefficients are less than one. In regime #2, a permanent increase in the real interest rate by 1 percentage point will *ceteris paribus* lead to an increase in the (expected) dividend-price ratio by 0.09 percentage point in the long run. Thus, only 9% (rather than the 100% implied by the theoretical framework) of the change in the real interest rate shows up in the dividend-price ratio. The impact of the risk premium is somewhat higher, as an increase in the premium by 1 percentage point raises the expected long-run dividend-price ratio by 0.16 percentage point in regime #1 (an impact of 16%) and by roughly the same magnitude in regime #2 (20%). A possible explanation is, like before, that market participants expect a significant part of the shocks to the two variables to be transitory. For the real interest rate, this may in particular be true in regime #1 (covering the subperiods 1927-1949 and 1986-1991, cf. below) where the real interest rate is subject to very large fluctuations, implying that a relatively large portion of the variation in the current real interest rate is transitory (see Figure 2). This could potentially explain the low and insignificant impact of the real interest rate in regime #1.
The level of real dividends has a significant positive impact on dividend-price so that stock prices appear to underreact to shocks to dividends, as compared to theory. A prime candidate for explaining this feature is, again, that shocks to dividends are expected, to some extent, to be transitory. Because we measure dividends in log-levels, the coefficients can be interpreted as semielasticities. From (7), a permanent *1%* relative increase in real dividends will in the long run lead to an increase in the (expected) dividend-price ratio by approximately *0.03 percentage point* in both regimes.

< Figures 11 and 12 >

The fit of the regime-switching model is illustrated in Figure 11, while Figure 12 shows the standardized residuals. Both the fit and the residuals are calculated using the filtered probabilities for the latent state variable s_t. The fitted (or expected) dividend-price ratio is calculated across regimes as[19]:

$$(8) \qquad E\left[\frac{D_t}{P_t}\right] = E\left[\frac{D_t}{P_t}|s_t = 1\right] * p_1^f + E\left[\frac{D_t}{P_t}|s_t = 2\right] * p_2^f$$

$p_i^f \equiv Pr\{s_t = i|I_T\}\ (i=1,2)$ is the filtered probability of state *i* at time t, conditional on the information set I_T which contains all available information on observables (including dividend-price) in the sample, cf. Hamilton (1994, Section 22). The state-conditioned means $E[.|s_t]$ follow immediately from (6), using the fact that the residual term has a zero mean. The variance of dividend-price around its fitted value, $VAR(D_t/P_t) \equiv E[D_t/P_t - E(D_t/P_t)]^2$, can be derived by using a formula similar to (8) for the second moment $E(D_t/P_t)^2$ and exploiting the fact that $E((D_t/P_t)^2|s_t) \equiv \sigma(s_t)^2 + (E((D_t/P_t)|s_t))^2$ (by the definition of variances). Subtracting the term $(E(D_t/P_t))^2$ (follows from (8)), then gives the variance. The result is:

$$(9) \qquad VAR\left(\frac{D_t}{P_t}\right) = p_1^f \sigma(1)^2 + p_2^f \sigma(2)^2 + p_1^f p_2^f \left(E\left[\frac{D_t}{P_t}|s_t = 1\right] - E\left[\frac{D_t}{P_t}|s_t = 2\right]\right)^2$$

The uncertainty of dividend-price is a result of both the unknown error term (captured by the first two terms in (9)) and the uncertainty arising from the fact that the state is unknown and the state-dependent means differ (the last term in (9)). The standardized residual which is a point estimate of the error term ϵ_t in (6) can, finally, be calculated as the difference between actual and fitted dividend-price, divided by the standard error of dividend-price (the square root of (9)).

[19] All moments in (8) and (9) and the following derivations are conditioned on the information set containing the past and current levels of the explanatory variables (including lagged dividend-price) as of period t (omitted for notational convenience).

Figures 11 and 12 show that the model captures the significant movements in dividend-price over most of the sample and, in particular, performs well in the beginning and towards the end of the sample. Like the one-regime model, the regime-switching model tracks the significant fall in dividend-price in 1983. Less appealing features are that the 1974 observation seems to be an outlier and that there are two subperiods (1947-1955 and 1958-1968) over which the model systematically underestimates, respectively, overestimates actual dividend-price.

The specification tests of Table 1 (bottom half) test whether the residual term of (6) is serially uncorrelated, homoskedastic and normal distributed. The tests reveal no misspecification at the standard 5% significance level, expect for the LM test for ARCH in regime #2. However, using an alternative small sample correction to that used in Table 1, the test for ARCH in regime #2 is (just) passed[20]. Notice that the tests for serial correlation, including the tests for serial correlation within the two regimes, are passed so that the tendency to a systematic under- and overestimation as noted above is not statistically significant. The Andrews (1993) test for structural break gives a test statistic close to its critical value at the 5% significance level. Hamilton (1996) suggests that a 1% significance level is used for this test in small samples to correct for a possible 'over-size', i.e., a tendency to indicate structural breaks too often in small samples. Using a 1% significance level, the test is passed with a comfortable margin, that is, we accept the null that the model has no structural breaks[21].

We conclude that the regime-switching model is, overall, well specified. In particular, the model performs well in regime #1. The Andrews test suggests that two regimes are sufficient to remove the structural breaks in the one-regime model.

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[20] Hamilton (1996) suggests two possible small sample corrections to the asymptotic Likelihood Ratio (LR) test; either to transform the test to a small sample version based on the F-distribution (which is the one used in Table 1) or to use a 1% significance level for the LR test. According to Monte Carlo simulations, both help in correcting for an 'over-size' of the specification tests (that is, the tests tend to indicate misspecification too often) in small samples. For the test for ARCH in regime #2, the LR test statistic which is asymptotically χ^2-distributed with 1 degree of freedom, is 6.17. The critical significance level is 1.3%. Hence, the null of no ARCH in regime #2 is (just) accepted at the 1% significance level.

[21] The individual LM statistics used in the Andrews test are reported in the Appendix. The test value is obtained for the year 1969.

According to the estimated transition probabilities, cf. Table 1, both regimes are highly persistent with the probability of continuing in a given regime being 96-97%. The state variable s_t is unobservable, but it is possible from the estimated transition probabilities and the estimated regime-dependent models to draw a probabilistic inference about the state for each year in the sample. This inference is expressed by the filtered state probability, that is, the probability of the economy being in (say) regime #1 in year t, conditional on all available sample information on observables (dividend-price and the economic fundamentals), cf. also above. The estimated filtered probabilities, expressed as the probability of regime #1, are shown in Figure 13. This plot confirms that the regimes are highly persistent. Furthermore, it gives a very clear inference about the state variable, suggesting that we can divide the sample into three distinct subperiods (using the 50% probability value as the dividing line between subperiods); 1927-1949, where the model of regime #1 governed the dividend-price process; 1950-1985 (regime #2), and 1986-1991 (regime #1). The identification of regimes corresponds quite well with the recursive plots of Figures 6-10 which, tentatively, indicate that there are two regime-shifts in the sample, one in the beginning and one towards the end.

< Figure 13 >

A further understanding of the two regimes can be facilitated by inspecting Figure 14 which shows the fit of each of the two regime-dependent models together with the actual dividend-price ratio. It is evident that the model of regime #1 systematically predicts a lower dividend-price ratio than that of regime #2 (over the sample, the average difference is 1.1 percentage point). This suggests that regime #1 (#2) is one with a low (high) dividend-price ratio and - correspondingly - a high (low) level of stock prices, taking due account of the underlying economic fundamentals. The recent period from 1986 where regime #1 has reigned could therefore be interpreted as a period with relatively high stock prices (even when taking account of fundamentals). The identification of regimes suggests that this period resembles the subperiod 1927-1949 in the beginning of the sample. The long intervening period from 1950 to 1985 has, on the other hand, been characterized by relatively low stock prices.

< Figure 14 >

The results from estimating the regime-switching model leads to the conclusion that the underlying economic model was subject to a structural break in both 1950 and 1986. The evidence that a regime-shift towards a lower dividend-price ratio occurred in the 1980s seems plausible given the large changes in the Danish economy in that period, cf. section 3. The timing of the regime-shift (1986) may be slightly surprising because the large adjustment in dividend-price as well as the structural changes took place in 1983. Thus, the economic model is able to explain the significant fall in dividend-price and the underlying increase in stock prices in 1983 without referring to a regime-shift. Using the estimated coefficients for the prevailing regime #2, the prime factor in explaining this event is the huge fall in the risk premium by about 11 percentage point that year which in itself explains a decline in dividend-price by 1.7 percentage point[22]. Moreover, a fall in the real interest rate (by nearly 12 percentage points) and real dividends (by 30% in relative terms) contribute an estimated 0.9 and 0.6 percentage point, respectively, to the decline in dividend-price. The regime-shift instead occurs in 1986. This shift is needed in order to explain why the dividend-price ratio remains low despite a reversal in the real interest rate, the risk premium and real dividends towards the levels prevailing before 1983. Tentatively, this lagged regime-shift might be consistent with the gradual phasing in of the pension fund tax. As a more general insight, the timing of the regime-shift in 1986 rather than 1983 highlights the importance of taking due account of underlying economic fundamentals when estimating whether or not a regime-shift has taken place.

The result that a regime-shift occurred in 1950 and that the pre-1950 regime should resemble that of the post-1986 period is harder to explain and a closer examination is needed.

It is evident from Table 1 that the allowance for two regimes significantly alters the estimated coefficients. The one-regime model does not come close to any of the regime-dependent models and we, in particular, encounter differences for the coefficients of real dividends and

[22] Recall that this large fall in the premium is partially motivated by the introduction of the new pension fund tax, cf. section 3, so that this particular variable incorporates one of the big structural changes in 1983. Also notice that within the regime-switching model, the pension fund tax may have induced a decline in the dividend-price ratio via two channels; by lowering the before-tax (risk) premium γ_t and, potentially, by triggering the shift to the 'low' dividend-price regime after 1986.

lagged dividend-price. The regime-switching model is better than the one-regime model in terms of fit (as measured by the likelihood or the estimated residual variances) which is no surprise as the regime-switching model contains more parameters. However, even after correcting for the number of parameters the regime-switching model seems superior. Table 1 shows the values of three information criteria often used as the basis for model selection; the Akaike information criterion (AIC), the Hannan-Quinn criterion (HQ) and the Schwarz criterion (SC). According to the first two, the regime-switching model is the preferable one, while the SC does not give a clear answer.

There are two more evident reasons for choosing the regime-switching model. First of all, the allowance for two regimes solves a clear problem with structural breaks in the one-regime model. Second, within the context of the regime-switching model, a one-regime model is valid if and only if the two regime-dependent models do not differ in any statistically significant way. This hypothesis can be put to a formal test by, for instance, testing whether all coefficients (including the constant term) are identical across the two regimes[23]. The Likelihood Ratio test of this null hypothesis gives a test statistic of 29.7 with 5 degrees of freedom, which corresponds to a critical significance level of 0.00%. Hence, the null is clearly rejected. We can conclude that there are two distinct regimes in the data and the evidence in favor of the regime-switching model is strong.

6. Conclusion

We have estimated a fundamentals-based economic model for the dividend-price ratio. Results show that our proxies for the growth-adjusted real interest rate and the risk premium on stocks are significant in explaining dividend-price empirically. This identification of a time-varying discount rate which is useful for empirical modeling is the main contribution of this paper. The existing empirical literature on modeling stock price behavior often ignores the time variation in the discount rate by assuming it to be constant. The estimated coefficients of the real interest rate and the risk premium are significantly less than one, the

[23] As noted by Hamilton (1990), it is not possible to perform a Likelihood Ratio (LR) test of the more adequate hypothesis that all parameters including the variances are identical across regimes. The reason is that the asymptotic information matrix becomes singular under the null because the transition probabilities can not be identified in the case of two identical regime-dependent models. This is a violation of one of the standard regularity conditions underlying the LR test.

value predicted by a Gordon-type theoretical model where all innovations in the two variables are expected by market participants to be permanent. This result suggests that the innovations are considered to be partially transitory. Lagged dividend-price and the level of real dividends are also important explanatory variables. The former captures slow adjustment in the dividend-price process while the significance of real dividends (with a positive coefficient) shows that stock prices tend to respond less than proportionately to dividend shocks. The latter may, again, reflect that market participants expect some of the shocks to dividends to be transitory.

We estimate the economic model using both a one-regime and a regime-switching approach. The latter is used to account for non-modeled changes in the exogenous ‘economic environment’, leading to structural changes in the economic model. Results show that it is important to allow for more than one regime over the sample in order to avoid structural breaks. Two regimes seem to suffice. The regimes correspond to two distinct versions of the economic model which differ wrt. the relative importance of the fundamentals variables. A main difference is that the real interest rate (proxy) is only significant in one of the regimes.

The two regimes also differ wrt. the level of the dividend-price ratio as one of the regimes has a systematically higher level for dividend-price than the other over the sample period considered. One way to interpret the two regimes is therefore to distinguish them as a ‘low-dividend-price’ regime (corresponding to a high level of stock prices) and a ‘high-dividend-price’ regime (low stock prices). The terms ‘high’ and ‘low’ are to be used within the context of the economic model which explicitly takes account of the underlying economic fundamentals. Results clearly identify three distinct subperiods (1927-1949, 1950-1985 and 1986-1991) in which the regimes (submodels) apply. The high persistence of the regimes gives plausibility to the working hypothesis that structural changes in the economy account for the shifts in the underlying economic model. The latest regime-shift in 1986 may possibly be explained by the gradual phasing in of a new separate tax on the returns on pension funds’ bond holdings, initiated in 1983.

Related literature is Driffill and Sola (1998) who estimate a two-state regime-switching model for the US stock market over the period 1900-1987, using the price-dividend ratio as the endogenous variable. Within the context of the standard Gordon model, they motivate the existence of two distinct states for price-dividend by the existence of two states of the underlying dividend process - a ‘low-growth-and-high-variance’ state and a ‘high-growth-and-low-variance’ state, respectively. These states result in two different fundamental solutions for the price-dividend ratio. Driffill and Sola (1998) find evidence of two states being present in the processes for dividends and price-dividend. Furthermore, they find that the allowance for two regimes leads to a significant improvement on the one-regime model, in particular, in terms of fit. Driffill and Sola (1998) also test for intrinsic bubbles in stock prices, as originally proposed by Froot and Obstfeld (1991), by allowing for the level of real dividends to explain price-dividend. Even though they cannot formally reject the existence of intrinsic bubbles, they conclude, based on the explanatory power of the models, that the inclusion of intrinsic bubbles is not important when first having allowed for two different regimes.

Our analysis differs from Driffill and Sola (1998) by using economic fundamentals, in particular, a time-varying real interest rate and a time-varying risk premium, in explaining dividend-price. Driffill and Sola (1998) focus exclusively on the regime-switching element, assuming a constant discount rate[24]. Our motivation for using the regime-switching approach is *ad hoc* in the sense that we do not provide a specific account of the causes of the regime-shifts. Driffill and Sola (1998), on the other hand, have a more firm theoretical foundation for the existence of distinct states in the pricing process, which is based on the existence of distinct states in the (exogenous) underlying dividend process.

The significance of real dividends in our analysis could - as in Driffill and Sola (1998) and Froot and Obstfeld (1991) - be suggestive of intrinsic bubbles in stock prices. However, this conclusion is only valid if certain restrictions on the parameters of the dividends and price processes are fulfilled, cf. Driffill and Sola (1998) and Froot and Obstfeld (1991). These have not been tested in the present paper.

The model we estimate provides a good fit to dividend-price, is overall well specified and does in particular work well in regime #1 (the periods 1927-1949 and 1986-1991). The model is not entirely satisfactory over subperiods in the middle of the sample (concentrated in regime #2) where we encounter a tendency to under-, respectively, overestimate dividend-price. The latter is a point where the model may be improved upon. Even though two regimes suffice according to formal testing, one possibility would be to allow for three regimes as the ‘problematic’ subperiods may be suggestive of an additional regime. However, allowance for a third regime increases the number of parameters to be estimated significantly (by 10).

The regime-switching model identifies regime-shifts in 1950 and 1986. An obvious but also challenging issue for future research is to identify the causes of the regime-shifts and, if possible, formally incorporate these as additional explanatory variables in the model. We have conjectured that the introduction of a new pension fund tax is a possible explanation of the regime-shift in 1986. By incorporating taxation in the economic model, the validity of this conjecture can be tested. Moreover, it would allow us to test whether changes in taxation also can account for the regime-shift in 1950. If so (and taxation is the sole explanation of the regime-shifts), the inclusion of taxation would remove the structural breaks from the underlying economic model.

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[24] We should add that the approach of Driffill and Sola (1998) is not applicable for Denmark as there is no evidence of distinct states in the process for dividends.

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Figure 1. Dividend-Price Ratio 1927-1996

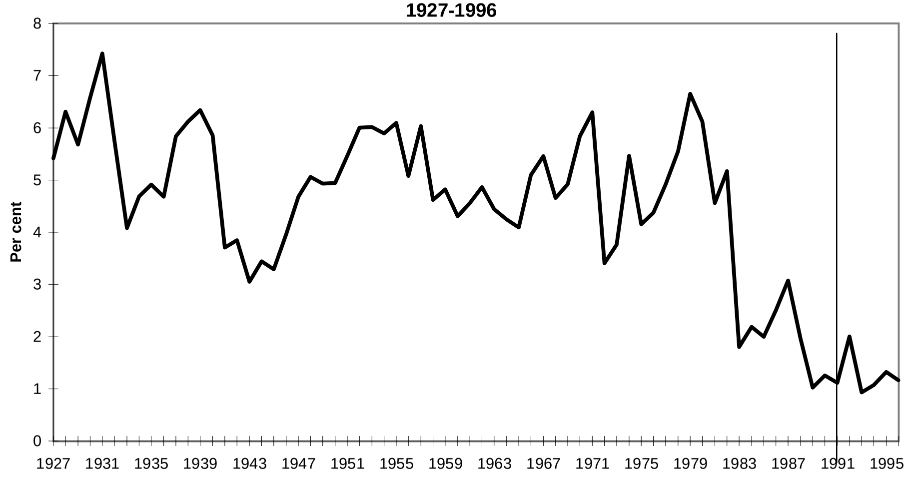


Figure 2. Growth-Adjusted Real Interest Rate 1927-1991

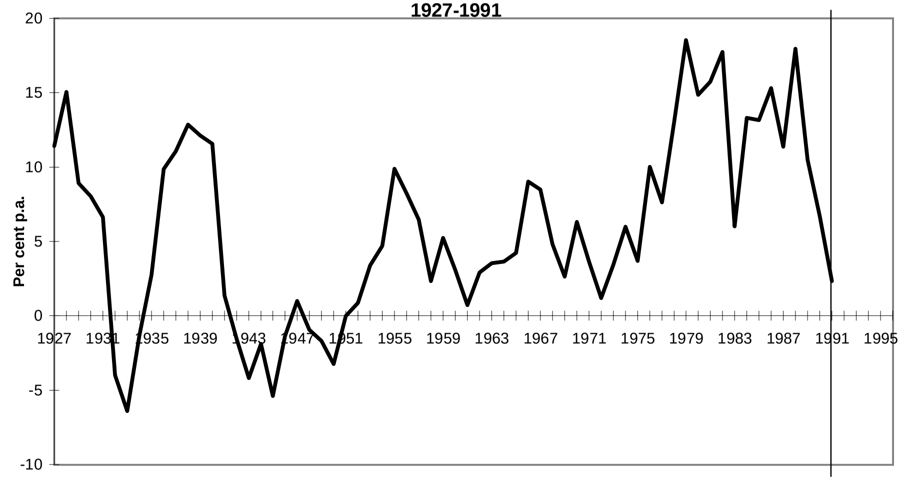


Figure 3. Risk Premium on Stocks 1927-1992

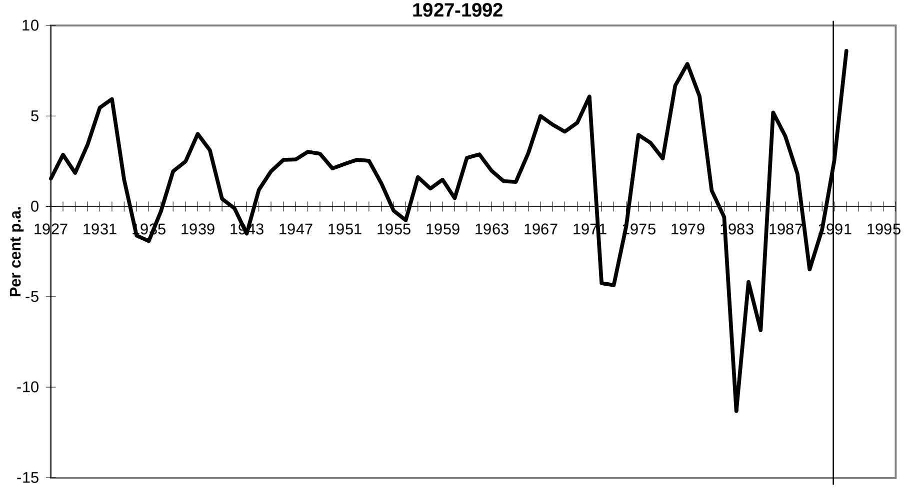
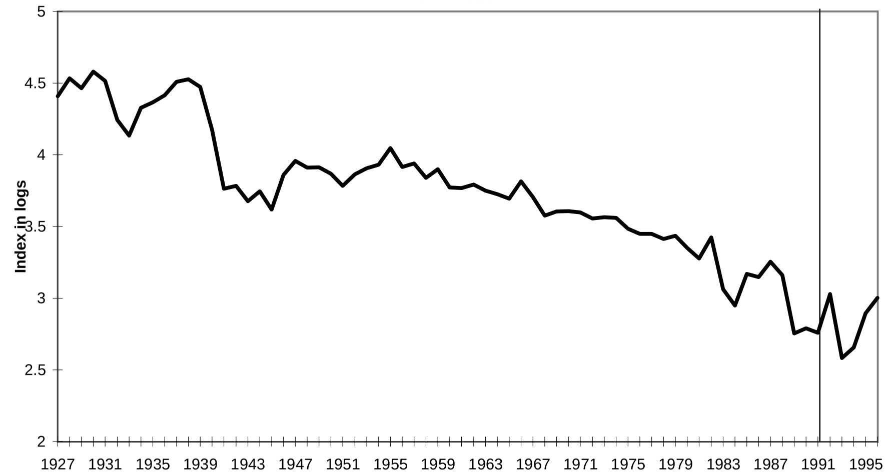
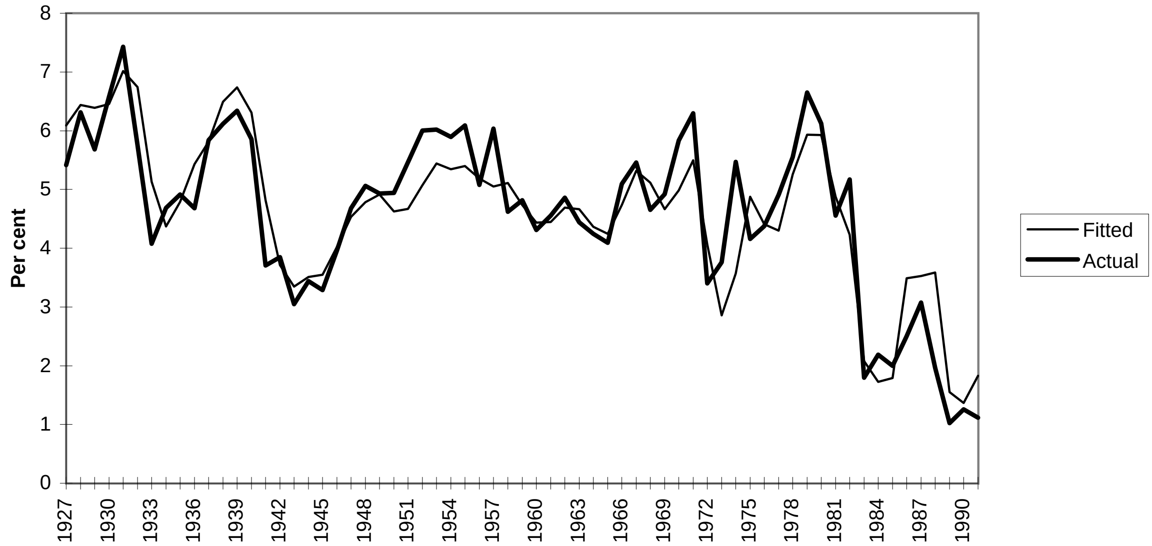


Figure 4. (Log-) Real Dividend Payments Per Share 1927-1996



Note: Vertical line marks end of estimation sample (1991).

Figure 5. Dividend-Price Ratio: Actual and Fitted One-Regime Model



Figures 6-10. Recursive Parameter Estimates for One-Regime Model

Recursive least squares point estimates (bold line) and 95% confidence band limits, 1942-1991. Sample start 1927.

Figure 6. Constant term

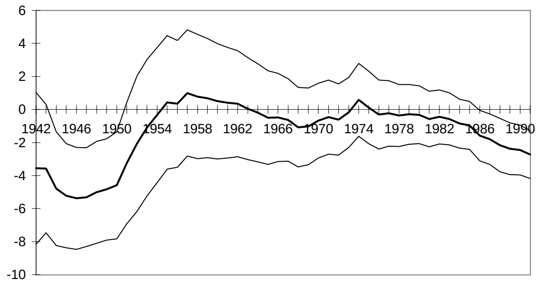


Figure 7. Real Interest Rate

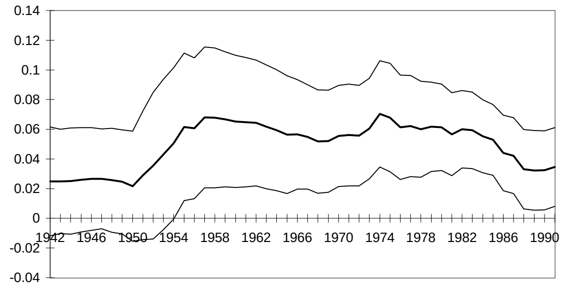


Figure 8. Risk Premium

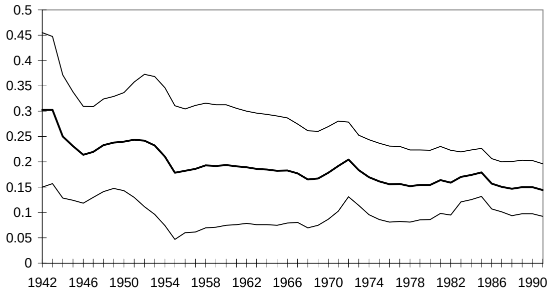


Figure 9. Real Dividends

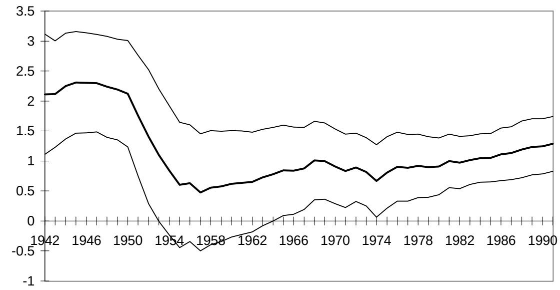


Figure 10. Lagged Dividend-Price Ratio

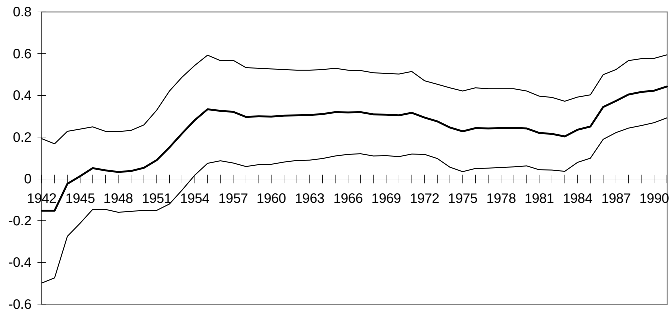


Figure 11. Dividend-Price Ratio: Actual and Fitted
Regime-Switching Model

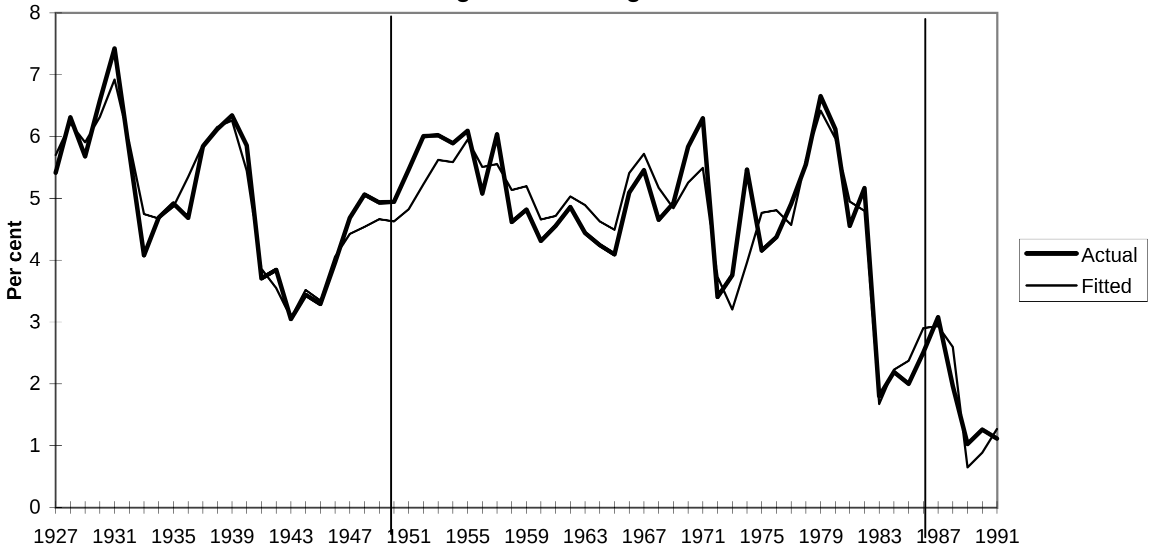
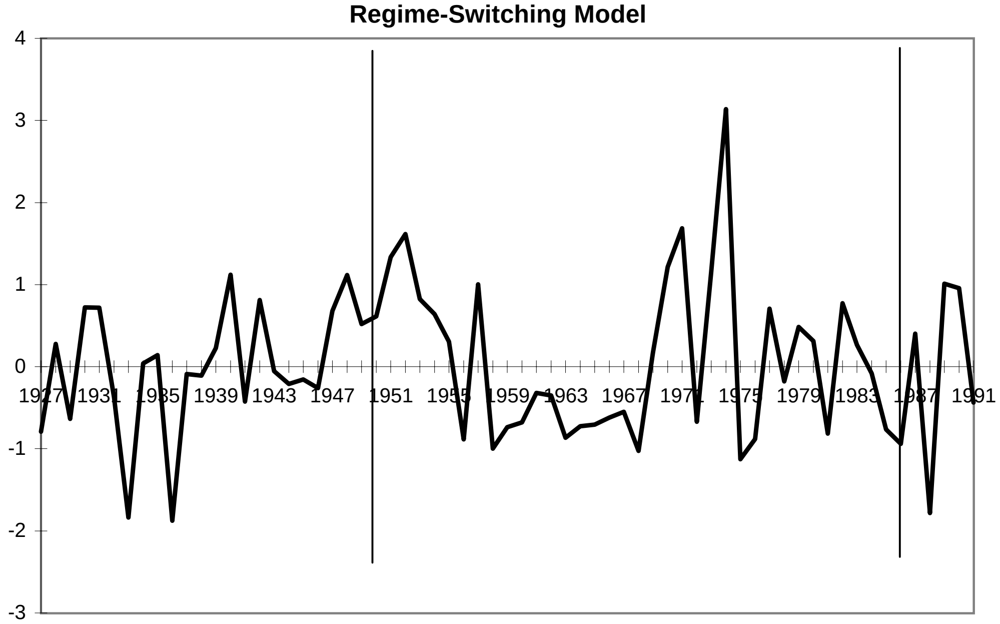


Figure 12. Standardized Residuals
Regime-Switching Model



<u>Note</u>: Vertical lines indicate datings of regime #1 (1927-1949 and 1986-1991) and regime #2 (1950-1985).

Figure 13. Filtered Probabilities for Regime #1

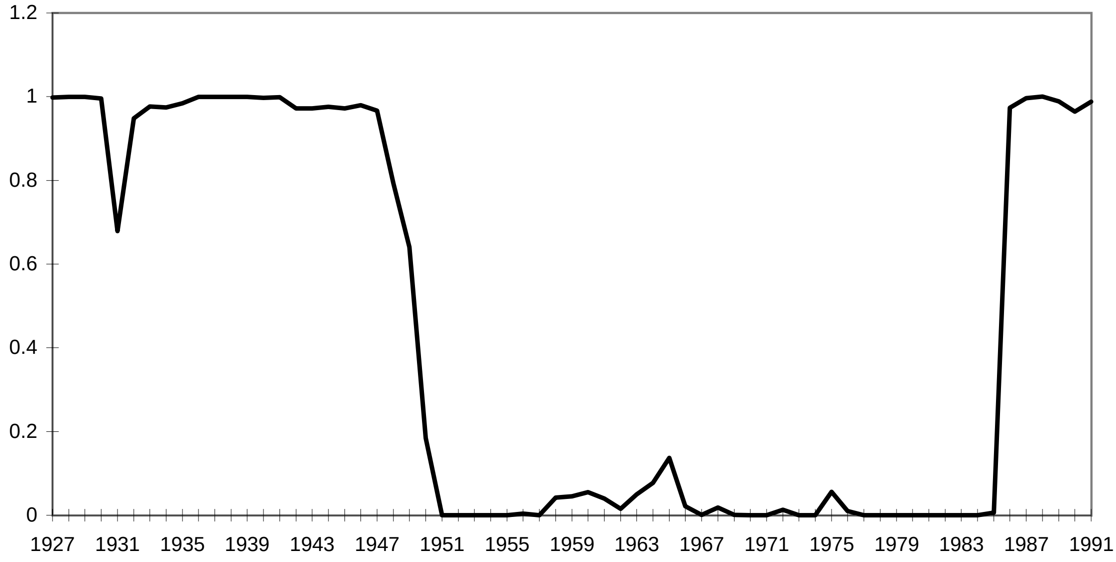
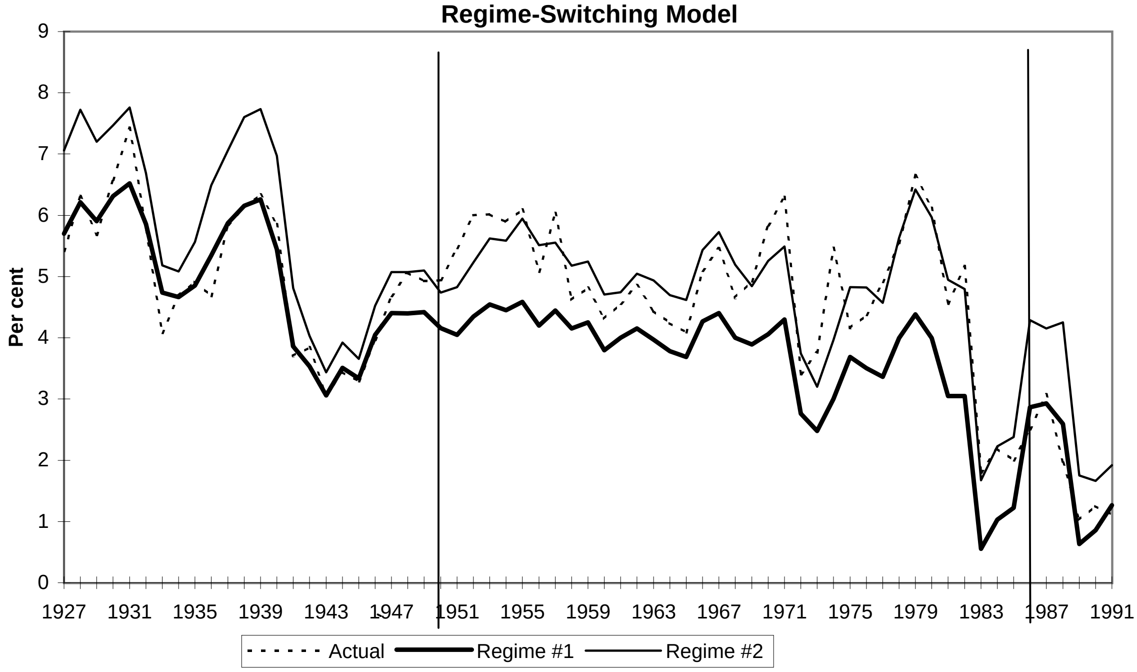


Figure 14. Dividend-Price Ratio: Actual and Regime-Dependent Predictions
Regime-Switching Model



Note: Vertical lines indicate datings of regime #1 (1927-1949 and 1986-1991) and regime #2 (1950-1985).

Table 1. Maximum Likelihood Estimates and Specification Testing: Models with and without Regime-Switching

		One-Regime Model	Regime-Switching Model	
Parameter estimates			Regime #1	Regime #2
Constant term	β_0	-2.7186 **	-5.8118 **	-4.8685 **
		(0.6976)	(0.6552)	(1.615)
Real interest rate	β_1	0.0345 **	0.0172	0.0725 **
		(0.0127)	(0.0112)	(0.0195)
Risk premium	β_2	0.1444 **	0.1399 **	0.1535 **
		(0.0250)	(0.0418)	(0.0242)
Real dividends	β_3	1.2848 **	2.3701 **	2.1473 **
		(0.2202)	(0.2414)	(0.4386)
Lagged D/P	β_4	0.4433 **	0.1143	0.2456 **
		(0.0728)	(0.0876)	(0.0913)
Variance	σ^2	0.3719	0.1255	0.2296
		(0.0652)	(0.0366)	(0.0525)
Transition probability	P_{ii}	—	0.9740	0.9626
			(0.0291)	(0.0295)
Ergodic probability		—	0.5901	0.4099
Log-likelihood		-60.0815	-43.7682	
AIC		132.2	115.5	
HQ		137.3	127.6	
SC		145.2	146.0	
White specification test 1)				
Autocorrelation	$F(4,51)$	2.737 (0.103)	2.333 (0.068)	
ARCH	$F(4,51)$	1.970 (0.166)	1.834 (0.137)	
Markov specification	$F(4,51)$	—	2.199 (0.082)	
LM specification test 1)				
Autocorr. regime #1	$F(1,51)$	—	0.529 (0.470)	
Autocorr. regime #2	$F(1,51)$	—	2.659 (0.109)	
Autocorrelation	$F(1,51)$	2.732 (0.104)	1.249 (0.269)	
ARCH regime #1	$F(1,51)$	—	2.493 (0.121)	
ARCH regime #2	$F(1,51)$	—	4.844 (0.032) *	
ARCH	$F(1,51)$	1.913 (0.172)	0.287 (0.595)	
Standardized residuals 1) 2)				
AR(1)	$F(1,63)$	3.106 (0.083)	0.863 (0.356)	
AR(3)	$F(3,61)$	5.519 (0.002) **	2.402 (0.076)	
AR(5)	$F(5,59)$	4.543 (0.002) **	1.403 (0.237)	
Normality	$\chi^2(2)$	2.810 (0.245)	3.775 (0.151)	
Andrews test for structural break 3)		23.009 **	8.964 *	

Note: Asymptotic standard errors of parameter estimates shown in parentheses, based on second derivatives of log likelihood function. A '*' shows significance at the 5% level, '**' at 1% level. The Akaike, Schwarz and Hannan-Quinn model selection criteria are calculated as: $AIC=-2l+2k$, $HQ=-2l+2ln(ln(T))k$, and $SC=-2l+kln(T)$, where l is the log-likehood value, k is the number of freely estimated parameters and T is the number of observations.

1) Test distributions apply to regime-switching model. For one-regime model, White and Lagrange Multiplier (LM) tests are distributed F(1,59). Tests are small-sample approximations based on the F-distribution, as suggested by Hamilton (1996). Critical significance levels in parentheses. The White and LM tests are described in Hamilton (1996).

2) For regime-switching model, the serial correlation (AR) tests are standard LM specification tests applied to a regression of the standardized residuals on a constant term. For one-regime model, standard LM tests on the regression equation. Normality test by Doornik and Hansen (1994).

3) Asymptotic critical test values are 8.85 (5% significance level) and 12.35 (1%), see Andrews (1993).

Appendix:

Andrews (1993) Tests for Structural Break

Figure A1. Individual LM Statistics Used in Andrews Test
One-Regime Model

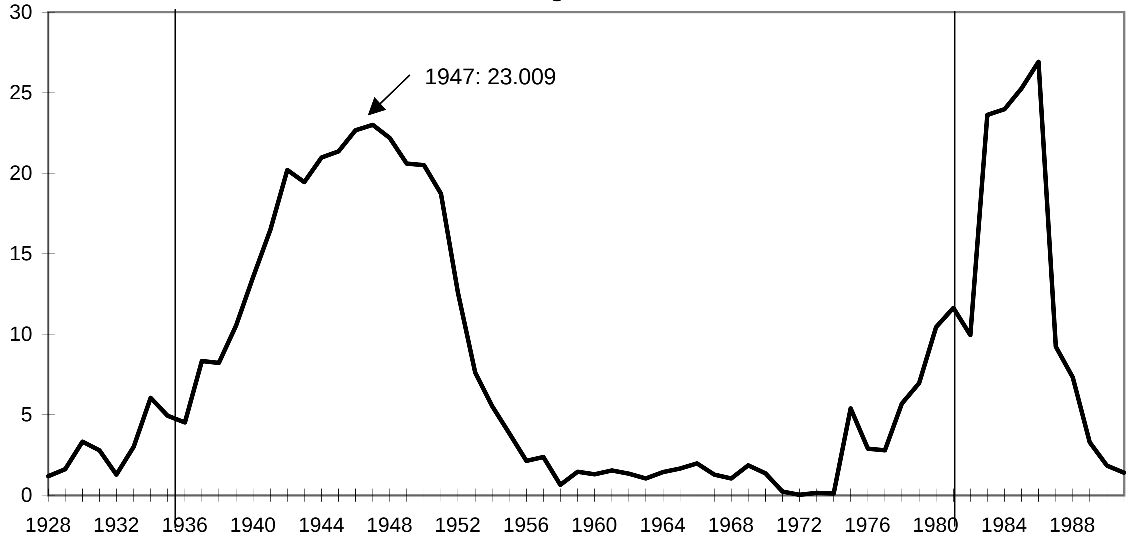
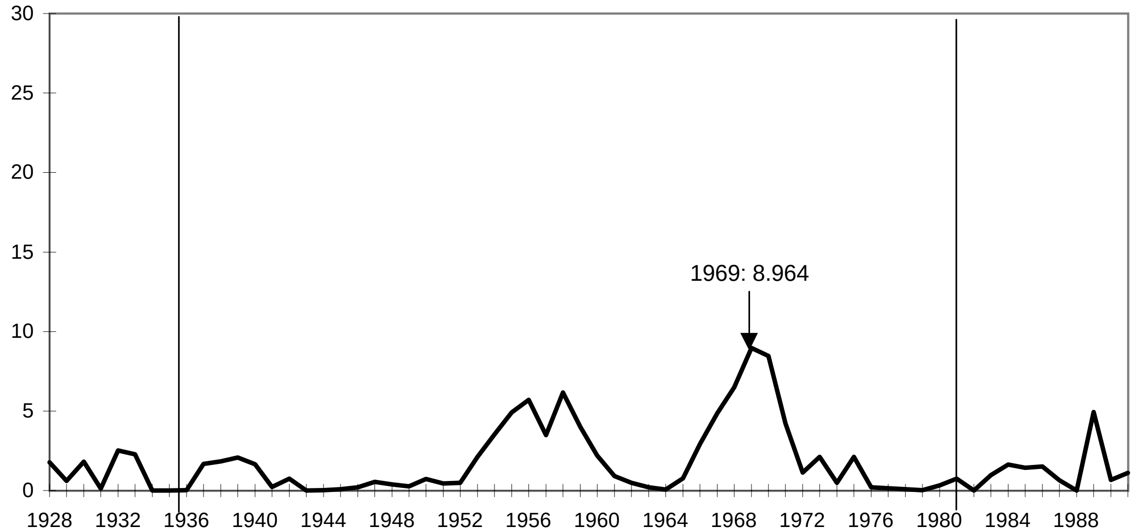


Figure A2. Individual LM Statistics Used in Andrews Test
Regime-Switching Model



Note: Only observations between 1936 and 1981 (both years included) are used in the Andrews test. Period indicated by vertical lines.